$\Omega = \{(1,1), (1,2), \ldots, (6,6)\}$: $6 \times 6 = 36$

$A = \{\text{doubles}\}$ 2, 4, 6, (8), 10

$B = \{\text{sum between 7 and 10}\}$ 12

$C = \{2, 7, 8\}$

$P(A) = 6/36 = 1/6$

$P(B) = (1,6), (6,1), (2,6), (6,2), (2,5),$
$(5,2), (3,6), (6,3), (3,5), (5,3),$
$(3,4), (4,3), (4,5), (5,4), (6,4)$
$(4,6), (5,5), (4,4)$

$P(A \cap B \cap C) = P(A)P(B)P(C)$

$P(B) = \frac{16}{36} = \frac{2}{4} = \frac{1}{2}$

$P(C) =$ (sum is 2) $(1,1)$, (sum is 7) 6 possibilities, $(3,5), (5,3), (2,6), (6,2),$

(4,4), 5/36 ← sum is 8

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$
$$= (1/6)(1/2)(12/36)$$
$$= (1/6)(1/2)(1/3)$$
$$= 1/36$$

1/36

A, B, C are mutually disjoint

if: $P(A \cap B \cap C) = P(A)P(B)P(C)$

(3) $P(A \cap C) = P(A)P(C)$

$P(A \cap B) = P(A)P(B)$

$P(B \cap C) = P(B)P(C)$

$P(A)P(C) = (1/6)(1/3)$

$P(A \cap C) = 1/18$

$P(\{(4,4),(1,1)\}) = 2/36$

$$P(B \cap C) \stackrel{?}{=} P(B)P(C)$$

$$\stackrel{?}{=} (1/2)(1/3) = 1/6$$

[redacted]

$$11/36 \neq 1/6$$

$$P(B \cap C) \neq P(B)P(C)$$

Problem 6

W is discrete RV, $\{1, 2, \ldots\}$

$$P(W > i+j \mid W > i) = P(W > j)$$

$$\frac{P(W > i+j \cap W > i)}{P(W > i)} = P(W > j)$$

$$\frac{P(W > i+j)}{P(W > i)} = P(W > j)$$

$$\star \quad P(W > i+j) = P(W > j)\,P(W > i)$$

$$P(W = 1) = p$$

$$P(W \neq 1) = 1 - p = q = P(W > 1)$$

Let's let $i = 1$. Using ☆

$$P(W > j+1) = P(W > j)\,P(W > 1)$$

$$= P(W > j)\,q$$

$$P(W > 1) = q$$

$$P(W > 2) = P(W > 1) \cdot q = q^2$$

$$P(W > 3) = P(W > 2+1) = P(W > 2) \times P(W > 1)$$

$$= q^2 \cdot q = q^3$$

$$P_n = P(W > n)$$

$$P(W > n) = P(W > n-1)\,P(W > 1)$$

$$P_n = P_{n-1} \cdot q$$

$$P(W > n) = q^n$$

$$P(W = n) ?$$

$$P(W = n) = P(W \leq n) - P(W \leq n-1)$$

$$1 - q^n - (1 - q^{n-1})$$