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Homework 2

1. Suppose that $P(A) = \frac{1}{3}$ and that $P(B^c) = \frac{1}{4}$. Can A and B be disjoint? Explain why or why not.

2. Suppose that a sample space S is finite and has n elements. Prove (combinatorial argument is acceptable) that the total number of possible subsets is 2^n.

3. Consider an experiment of tossing two dice, so the sample space Ω is

$$\Omega = \{(1,1),(1,2),\ldots,(6,6)\}.$$

   Define the following events:

$$\begin{aligned} A &= \{\text{doubles appear}\} = \{(1,1),(2,2),\ldots,(6,6)\}, \\ B &= \{\text{The sum is between 7 and 10}\}, \\ C &= \{\text{The sum is 2 or 7 or 8}\}. \end{aligned}$$

   (a) Find the following probabilities:
   - $P(A)$
   - $P(B)$
   - $P(C)$
   - $P(A \cap B \cap C)$.

   (b) Are A, B, C mutually independent? Why or why not.

4. Suppose that A and B are events such that $P(A), P(B) > 0$. Prove the following statements:

   (a) If A and B are mutually exclusive, they cannot be independent.

   (b) If A and B are independent, they cannot be mutually exclusive.

5. Let X and Y be random variables with cdf F_X and F_Y, respectively. X is said to be *stochastically greater than or equal to* Y if $F_X(t) \leq F_Y(t)$ for all t, (*stochastically greater* if there is at least some t_0 such that $F_X(t_0) < F_Y(t_0)$). Show that this implies $P(X > t) \geq P(Y > t)$ for all t. That is, X tends to be larger than Y.

6. Let W be a random variable taking the values in the set of integers $\{1, 2, \ldots\}$ with $P(W = j) > 0$ for all $j \geq 1$, and having the so-called "memoryless" property that

$$P(W > i + j \mid W > i) = P(W > j)$$

   Show that W is geometrically distributed.

   (Hint: W is a discrete random variable, and therefore must assign probability to each possible value of W. Call $P(W = 1) = p$, and $P(W > 1) = 1 - p$. For convenience, you may want to use a shorthand like $1 - p = q$).

7. (Banach match problem) Suppose that Jeremy has two pockets full of his favorite candy, each pocket with n pieces. Jeremy will randomly pick a pocket to take candy from (with equal probability each time). Continuing this practice, he will eventually run out of candy in one pocket; let X be a random variable representing the amount of candy in his *other* pocket the first time that he reaches into a pocket and finds it empty. Find the probability mass function for X.

   (Hint: choose one of the pockets to be called a "success", which is selected with probability p. If $X = k$, that means that he has picked a "success" $n - k$ times. What kind of distribution does this sound like? With this approach, don't forget that we arbitrarily selected one of the pockets to be a "success". How many ways can we choose which pocket is the "success"?)