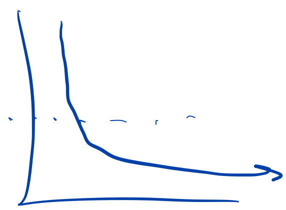
#6 $X \sim \text{pois}(\lambda)$; what value(s) of k maximize the pmf of X: $p(k)$.

$$p(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k \in \{0, 1, 2, \ldots\}, \quad \lambda > 0.$$

$$f(k) = \frac{p(k)}{p(k-1)} = \frac{\left(\frac{\lambda^{(k-1)+1} \cancel{e^{-\lambda}}}{k!}\right)}{\left(\frac{\cancel{\lambda^{(k-1)}} \cancel{e^{-\lambda}}}{(k-1)!}\right)} = \frac{\lambda}{k}$$



if $f(k) = 1$: $\frac{p(k)}{p(k-1)} = 1$ or $p(k) = p(k-1)$.

in this case, no change

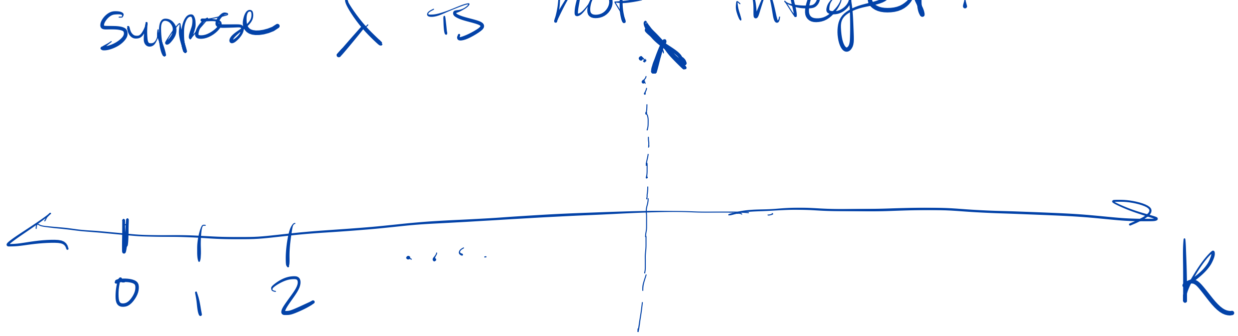
if $f(k) > 1$: $\frac{p(k)}{p(k-1)} > 1$, and $p(k) > p(k-1)$.

in this case, we increase when going from $k-1 \to k$.

if $f(k) < 1$: we are now decreasing

$$f(k) > 1 \iff \frac{\lambda}{k} > 1 \iff k < \lambda \in \mathbb{R}$$

suppose λ is not integer.

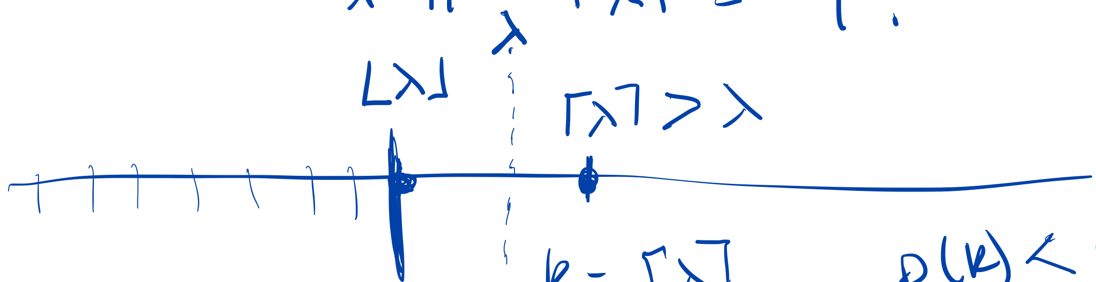


$\lfloor \lambda \rfloor$ = floor (λ) : biggest int. smaller than λ.

eg: $\lambda = \pi$, $\lfloor \lambda \rfloor = 3$

$\lceil \lambda \rceil$ = ceil (λ) : smallest int. bigger than λ

$\lambda = \pi$, $\lceil \lambda \rceil = 4$.



$k = \lceil \lambda \rceil$, $\underbrace{p(k) < p(k-1)}$

if λ is not whole number,

then p(k) is maxed at $\lfloor \lambda \rfloor$.

$\frac{p(k)}{p(k-1)}$

$\lambda \in (0,1)$.

$k = \lfloor \lambda \rfloor = 0$

p(k) = p.m.f. prob. mass function of $X \sim \text{pois}(\lambda)$.

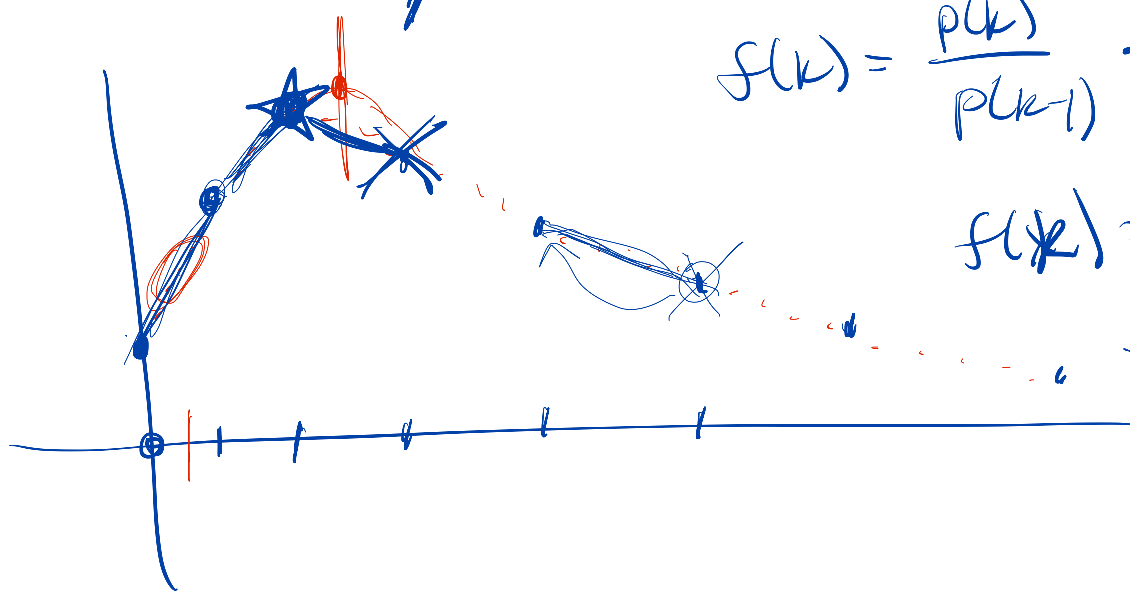
$\frac{p(0)}{p(-1)}$

$\frac{p(1)}{p(0)} < 1$

$X \sim$ number of fish in a lake for a given time.

$X \sim \text{pois}(\lambda)$

~~that~~



$$f(k) = \frac{p(k)}{p(k-1)} < 1$$

$$f(k) = \frac{p(k)}{p(k-1)} > 1$$

true for

$k=1, k=2$

$k=3$

$f(k) < 1$, decr