YOUR NAME
YOUR EMAIL
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Homework 3

1. Show that the binomial probabilities sum to 1.

2. If X is an integer valued random variable, show that the pmf is related to the cdf by:

$$p(k) = F(k) - F(k-1).$$

3. Let A and B be events, and let I_A and I_B be the associated indicator random variables. That is, $I_A = 1$ if event A occurs, and $I_A = 0$ otherwise. Show that:

$$I_{A\cap B} = I_A \times I_B = \min(I_A, I_B),$$

and

$$I_{A\cup B} = \max(I_A, I_B).$$

4. Which is more likely: 9 or more heads in 10 tosses of a fair coin, or 18 or more heads in 20 tosses of the same coin?

5. Suppose that I receive phone calls as a Poisson process with parameter $\lambda = 2$ per hour.

   (a) If I take a 10-min shower, what is the probability that the phone rings during that time?

   (b) How long can my shower be if I want to ensure that the probability of receiving no calls to be at most 0.5?

6. Let $X \sim \text{Poisson}(\lambda)$. What value of k maximizes the pmf of X, $p(k)$?

   (Hint: consider the ratio $p(k)/p(k-1)$. Use this to derive when the function is increasing and when the function is decreasing. Don't forget that $k \in \{0, 1, 2, \ldots\}$).