problem 1, part b.

$F(x) = 1 - e^{-\alpha x^{\beta}}$, $\underline{x \geq 0}$ $\qquad f(x)$: pdf.

$\underbrace{F(x) = \int_{-\infty}^{x} \underbrace{f(x)\,dx}}$ $\leftarrow$ F.T.C $\Rightarrow$

$$\frac{d}{dx} F(x) = f(x)$$

$$\frac{d}{dx}\left(1 - e^{-\alpha x^{\beta}}\right) = -e^{-\alpha x^{\beta}} \cdot \frac{d}{dx}\left(-\alpha x^{\beta}\right) = e^{-\alpha x^{\beta}} \cdot \alpha \beta x^{\beta - 1}$$

$$\begin{cases} = \underline{\alpha \beta x^{\beta-1} e^{-\alpha x^{\beta}}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

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(1):

- $\lim_{x \to -\infty} F(x) = 0 \qquad \lim_{x \to \infty} F(x) = 1$
- $F(x)$ is non-decreasing $\rightarrow$ <u>monotonically increasing</u>
- $F(x)$ is <u>right-continuous</u>.

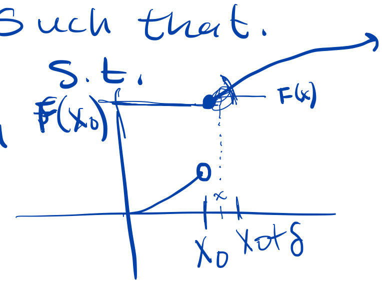
- <u>Right continuity</u>: $\quad x_0$.

$$\left.\begin{array}{l} \lim_{x \to x_0^+} F(x) = F(x_0) \\ \lim_{x \downarrow x_0} F(x) = F(x_0). \end{array}\right\}$$

$\forall$ for all $\varepsilon > 0$, $\exists$ there exists $\delta > 0$, such that. s.t.

for all $x_0 < x < x_0 + \delta$, then

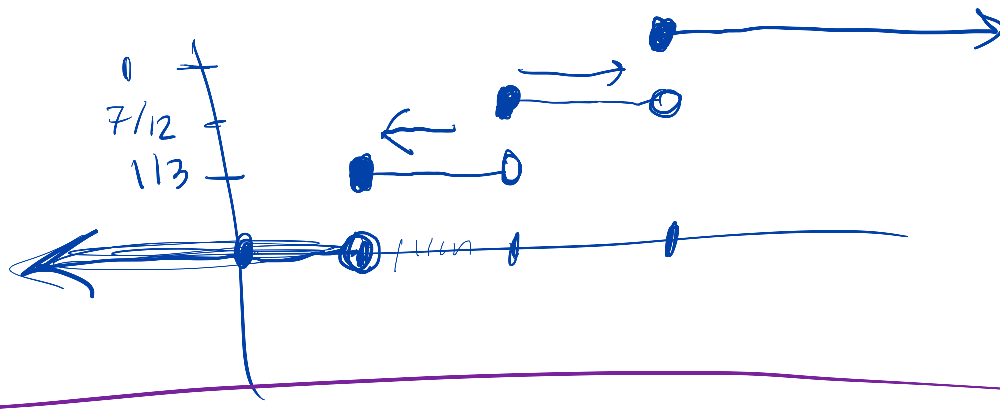
$$|F(x) - F(x_0)| < \varepsilon$$



- discrete R.V.

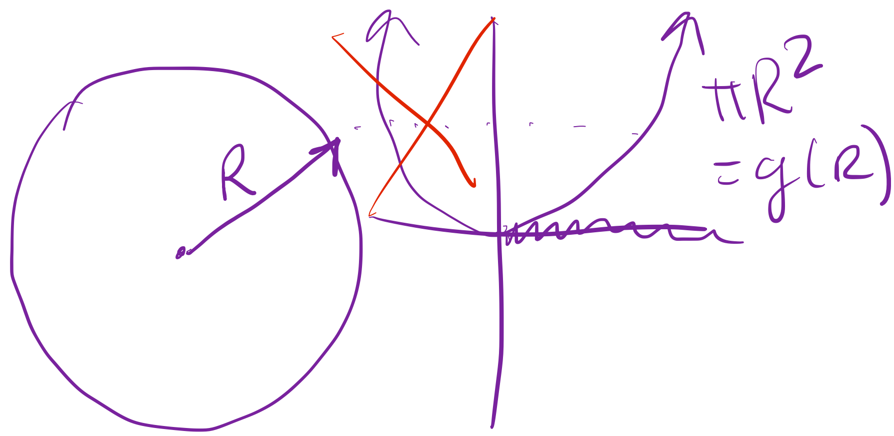
$$X = \begin{cases} 1 & 1/3 \\ 2 & 1/4 \\ 3 & 5/12 \\ 0, & \text{o.w.} \end{cases}$$

$F_X(x) = P(X \leq x)$



problem 6:

$R \sim \exp(\lambda)$



A = Area of the produced circle

$A = \pi R^2$ $\quad g^{-1}(a)$

$$f_A(a) = f_R(g^{-1}(a)) \cdot \left| \frac{d}{da} g^{-1}(a) \right|$$

$$\underset{F_A(a)}{P(A \leq a)} = P(\pi R^2 \leq a) = P\left(R \leq \sqrt{\frac{a}{\pi}}\right)$$

$$= F_R\left(\sqrt{\frac{a}{\pi}}\right)$$

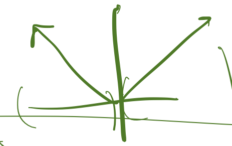
$$F_A(a) = P\left(R \le \sqrt{\frac{a}{\pi}}\right) = F_R\left(\sqrt{\frac{a}{\pi}}\right)$$

$$f_A(a) = \frac{d}{da} F_A(a) = \underbrace{\frac{d}{da} F_R\left(\sqrt{\frac{a}{\pi}}\right)}$$

$$= f_R\left(\sqrt{\frac{a}{\pi}}\right) \cdot \frac{d}{da}\left(\sqrt{\frac{a}{\pi}}\right)$$

$$\int_{u}^{u} c e^{-\frac{1}{2}(x-u)^2} dx$$

#4: $X \sim N(0, \sigma^2)$, $Y = |X|$.



$Z \sim N(0,1)$ $\quad F_Z(z) = P(Z \le z) = \underbrace{\Phi(z)}$

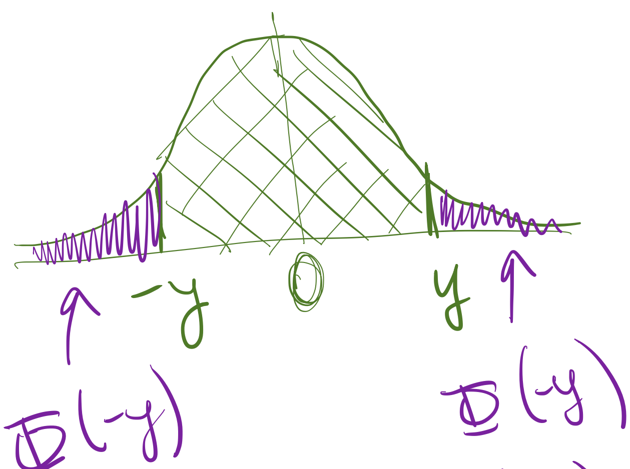
$F_X(x) = P(X \le x) = P(\sigma Z \le x) = P\left(Z \le \frac{x}{\sigma}\right) = \underline{\Phi\left(\frac{x}{\sigma}\right)}$

$X \sim N(0, \sigma^2) \iff \sigma Z \overset{d}{=} X, \; Z \sim N(0,1)$

$F_Y(y) = P(Y \le y) = P(|X| \le y)$

$= P(-y \le X \le y)$

$= 1 - 2\Phi\left(\frac{y}{\sigma}\right)$



$1 - 2\Phi(-y)$

$f_Y(y) = \frac{d}{dy} F_Y(y)$

$= \frac{d}{dy}\left(1 - 2\Phi\left(\frac{y}{\sigma}\right)\right)$

chain-rule

$\frac{d}{dx}\Phi(x) = \boxed{\phi(x)}$

cdf - standard normal

pdf - standard normal

$$\boxed{\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}}$$