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Homework 4

1. Let $\alpha, \beta > 0$, and define the function $F(x)$ to be:

$$F(x) = \begin{cases} 1 - e^{-\alpha x^{\beta}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

   (a) Show that F is a cdf for some random variable. (Hint: Theorem 2.1).
   (b) Calculate the corresponding pdf.

2. Let $\alpha \in [-1, 1]$, and define the function f to be

$$f(x) = \begin{cases} \frac{1+\alpha x}{2} & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that f is a density for some continuous random variable. (Hint: Theorem 2.2. This theorem just gives properties of any pdf; the fact that it is tied to some random variable is a result of the next part of this question, and Theorem 2.1).
   (b) Find the corresponding cdf.

3. For some value c, suppose that X has the density $f(x) = cx^2$, with support $\mathcal{X} = [0, 1]$ (i.e., $f(x) = 0$ if for $x \notin [0, 1]$).

   (a) Find the value of c.
   (b) What is $P(.1 \leq X < 0.5)$.

4. If $X \sim N(0, \sigma^2)$, find the density of $Y = |X|$.

5. Let $X \sim N(\mu, \sigma^2)$. Find the density of $Y = e^X$ (note: this is called the log-normal density, since $\log(Y) = X$ is normally distributed).

6. If the radius of a circle is modeled as an exponential(λ) random variable, find the pdf of the random variable A that represents the area of the corresponding circle.

7. Theorem 2.3 from our lecture slides states that if X is a random variable with CDF $F_X(x)$, and if g is a strict, monotone function, then if $Y = g(X)$:

   (i) If g is increasing, then $F_Y(y) = F_X\big(g^{-1}(y)\big)$,
   (ii) If g is decreasing, then $F_Y(y) = 1 - F_X\big(g^{-1}(y)\big)$.

   Part (7ii) was shown in class when X is a *continuous* random variable. Your task is the following:

   - Assume that X is a continuous random variable. **Show that Theorem 2.3 holds when g is a strictly monotonically increasing function**. That is, show that the statement (7i) is true.