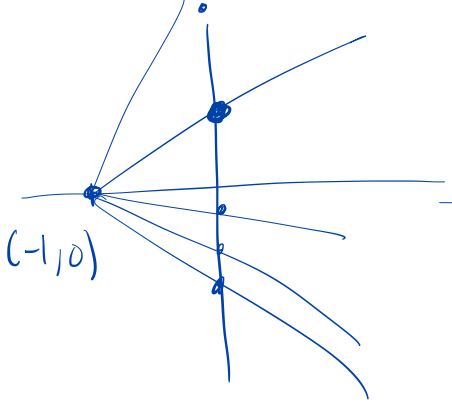
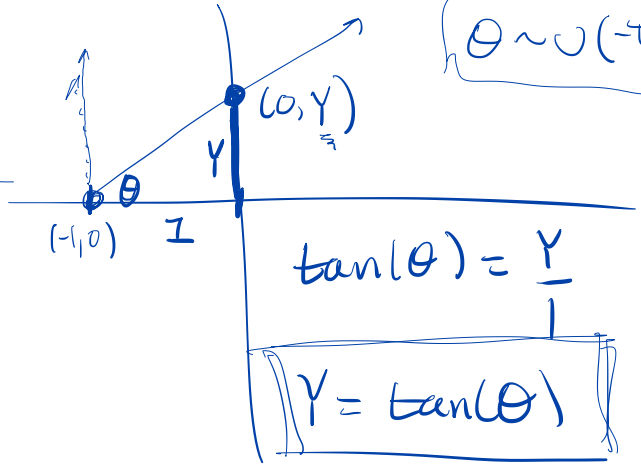
#4



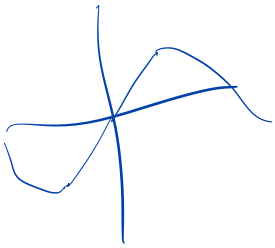


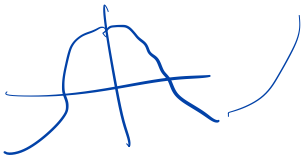
$\theta \sim U(-\pi/2, \pi/2)$

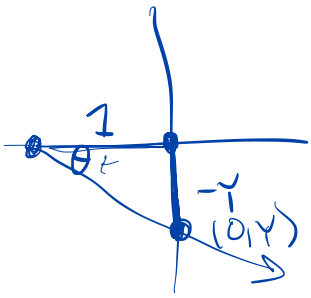
$\tan(\theta) = \frac{Y}{1}$

$Y = \tan(\theta)$

$f_Y(y)$







~~$\theta > 0$~~

$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$

$= \frac{-\sin(\theta)}{\cos(\theta)}$

$= -\tan(\theta)$

$\theta \sim U(-\pi/2, \pi/2)$

$Y = \tan(\theta)$

#5 $X \sim \exp(\lambda)$

$\lfloor X \rfloor$: floor of X, an integer

$X - \lfloor X \rfloor = Y$: the part of X that is lost when doing floor. $Y \in [0, 1)$

$X(\omega) = 3.8741$, $\lfloor X(\omega) \rfloor = 3$

$X - \lfloor X \rfloor = 0.8741$

$X(\omega) = 4$, $X - \lfloor X \rfloor = 4 - 4 = 0$

let $k \in \mathbb{Z}^+$

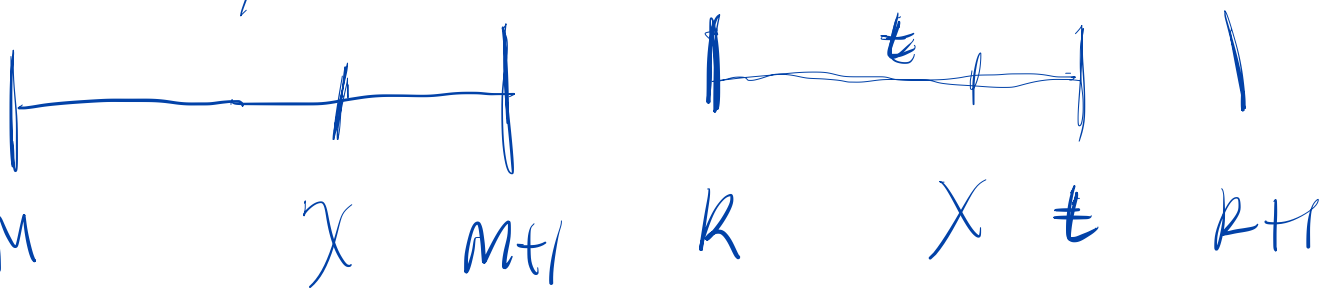
$$P(\lfloor X \rfloor = k) = P(k \le X < k+1)$$

$$= \int_k^{k+1} f_X(x)\,dx$$

$$= F_X(k+1) - F_X(k)$$

5(b): $P(\lfloor X \rfloor = m, Y \le t)$

$= P(m \le X < m+1, X - \lfloor X \rfloor \le t)$



$= P(m \le X \le m+t)$

$P(Y \le t) = \sum_m P(\lfloor X \rfloor = m, Y \le t)$