YOUR NAME
YOUR EMAIL
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Homework 5

1. (1 point) Let $U \sim U(0,1)$. For some $\lambda > 0$, let $g(x) = -\frac{1}{\lambda}\ln(1-x)$. What is the distribution of $Y = g(U)$?

2. (1 point) Suppose that $X \sim N\big(1, 2^2\big)$ (i.e., $\sigma^2 = 4$). Find the density of $Y = X^3$.

3. (2 points) Let $X \sim \text{Exp}(1)$, and $Y = (X-1)^2$. Find the density function of Y.

4. (2 points) Consider the following experiment. An individual creates a standard cartesian X-Y plane (meaning we have a standard X-Y coordinate grid), and places their pencil at the point $(-1, 0)$. Then, they draw a line towards the y-axis at a (uniform) random angle Θ, such that $\Theta \sim U(-\pi/2, \pi/2)$. In doing so, their line will eventually cross the y-axis, at some random point $(0, Y)$. Find the pdf of the random variable Y, which represents where the line crosses the y-axis.

   - (Hint): Draw a picture of what's happening. Try a few different "random" lines from the point $(-1, 0)$ to the y-axis. Your solution will involve some Trigonometry.

5. (Challenging) Let X follow a Exponential(λ) distribution, and denote $\lfloor X \rfloor$ as the "floor" of X, or the greatest integer not exceeding X (e.g., $\lfloor \pi \rfloor = 3$).

   Now let $Y = X - \lfloor X \rfloor$. Note that Y is a continuous random variable, that takes on values in $[0, 1)$.

   (a) (1 point) Find the pmf of $\lfloor X \rfloor$.

   - (Hint): Work from first principles, don't try the change of variables formula (though it works, just much harder). Because $\lfloor X \rfloor$ is discrete, we want to find $P(\lfloor X \rfloor = k)$ for interger values k. What does this mean for X?
   - (LaTeX Hint): For those wanting to LaTeX, I had to create a new command `\floor{}`. How this command is defined can be found in the header of the generating tex file.

   (b) (2 points) Find the pdf of Y.

   - (Hint): Consider finding $P(\lfloor X \rfloor = m, Y \le t)$, using a similar approach as above. Once that is done, then calculate

$$P(Y \le t) = \sum_m P(\lfloor X \rfloor = m, Y \le t)$$