YOUR NAME
YOUR EMAIL
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Homework 6

1. (1 point) If X and Y are independent exponential(λ) random variables, find the joint density of the polar coordinates R and Θ of the point (X, Y). Are R and Θ independent?

2. (1 point) Let X and Y be jointly continuous random variables, with pdf $f(x, y)$. Find an expression for the density of
$$Z = X - Y.$$

3. (1 point) If T_1 and T_2 are independent exponential random variables, find the density function of
$$R = T_{(2)} - T_{(1)}.$$

4. (1 point) If X_1 and X_2 are independent Poisson random variables with parameters λ_1 and λ_2 respectively, then show that $S_2 = X_1 + X_2$ is a Poisson random variable with parameter $\lambda_1 + \lambda_2$.

5. (1 point) Use the problem above and induction to argue that if X_i for $i = 1, 2, \ldots, n$ are independent Poisson random variables with parameters λ_i, respectively, then if we define the sum as
$$S_n = X_1 + X_2 + \ldots + X_n,$$
then S_n is a Poisson random variable with parameter $\sum_i \lambda_i$.

6. Let T be an exponential random variable with parameter β and let W be a random variable independent of T which assumes the value 1 with probability 2/3 and the value -1 with probability 1/3.
   (a) (1 point) Find the density of $X = WT$.
   Hint: I suggest working from first-principles here. Consider $F(x) = P(X \leq x)$, and split up the event $\{X \leq x\}$ as the union of $\{X \leq x, W = 1\}$ and $\{X \leq x, W = -1\}$.
   (b) (1 point) Find $E[X]$

7. A random variable V is said to have a distribution symmetric about 0 if the distriubtion of V is the same as that of $-V$. Let V be a continuous random variable with continuous density function f.
   (a) (1 point) Show that V is distributed symmetrically about 0 if and only if $f(t) = f(-t)$, for every t. In other words, f is an even function.
   (b) (1 point) Show that $E(V) = 0$.