YOUR NAME
YOUR EMAIL
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Homework 10 (Final Exam Review)

Note: This homework will only be graded on submission. It's purpose is to help prepare you for the final exam.

1. (1.8.9, Rice) The weather forecaster says that the probability of rain on Saturday is 20% and that the probability of rain on Sunday is 25%. Is the probability of rain during the weekend 50%? Why or why not?

2. (1.8.12, Rice) In a game of poker, five players are each dealt 5 cards from a 52-card deck. How many ways are there to deal the cards?

3. (2.5.39, Rice) The Cauchy cumulative distribution function is

$$F(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x), x \in \mathbb{R}.$$

   (a) Show that this is a CDF.

   (b) Find the corresponding density function.

   (c) Find x such that $P(X > x) = 0.1$.

4. (3.8.18, Rice) Let X and Y have the joint density function $f(x, y) = k(x - y)$, for $0 \leq y \leq x \leq 1$.

   (a) Sketch the region over which the density is positive and use it in determining limits of integration to answer the following questions.

   (b) Find k.

   (c) Find the marginal densities of X and Y.

   (d) Find the conditional densities of Y given X and X given Y.

5. Let $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$. First using a change of variables approach (CDF method or change of variables formula), find the density of Y. Next, find the denisty of Y by first considering the MGF of Y, and then matching that to the MGF of a known distribution.

6. (3.8.69, Rice) Find the density of the minimum of n independent Weibull random variables, each of which has the density

$$f(t) = \beta\alpha^{-\beta}t^{\beta-1}e^{-(t/\alpha)^{\beta}}.$$

7. (4.7.25, Rice) If X_1 and X_2 are independent random variables following a gamma distribution with parameters α and λ, find $E(R^2)$, where $R = X_1^2 + X_2^2$.

8. (4.7.49, Rice) Two independent measurements, X and Y, are taken of a quantity μ, such that $E[X] = E[Y] = \mu$. Denote $\text{Var}(X) = \sigma_X^2$, and $\text{Var}(Y) = \sigma_Y^2$. The two measurements are combined by means of a weighted average to give

$$Z = \alpha X + (1 - \alpha)Y,$$

   where $0 \leq \alpha \leq 1$.

   (a) Show that $E(Z) = \mu$.

   (b) Find α in terms of σ_X and σ_Y to minimize $\text{Var}(Z)$.

   (c) Under what circumstances is it better to use the average $(X + Y)/2$ than either X or Y alone?

9. Let Y have a density which is symmetric about 0 and let $X = SY$, where S is independent of Y and assumes values 1 and -1 with probability 1/2. Show that $\text{Cov}(X, Y) = 0$ but that X and Y are *not* independent. (This shows that uncorrelatedness does not necessarily imply independence.)

10. Let (U, V) be distributed jointly with a spherically symmetric density. In other words, let their joint density $f(u, v) = Cg(u^2 + v^2)$, for some non-negative function g. Show that $(\epsilon_1 U, \epsilon_2 V)$ has the same distribution as (U, V), where ϵ_1 and ϵ_2 are either 1 or -1. Deduce that U and V are uncorrelated.

11. Let (X, Y) have a joint uniform distribution inside the ellipse given by $\{(x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\}$. Find the marginal densities of X and Y and the conditional densities of Y given X and X given Y. Are X and Y correlated? Are they independent?

12. (5.4.15, Rice) Suppose that you bet \$5 on each of a sequence of 50 independent fair games. Use the central limit theorem to approximate the probability that you will lose more than \$75.

13. Let $X_1, X_2, \ldots, X_n$ be iid random variables with $E[X_i] = 2$, and $\text{Var}(X_i) = 4$. Using the CLT and the Delta-method, get an approximate distribution for $\left(\bar{X}_n\right)^{4/3}$ if $\bar{X}_n$ is the average of the variables $X_1, \ldots, X_n$.