Math 4450 F25: Midterm Formulas

- **Poisson Distribution**. If X has a Poisson(λ) distribution, then the pmf of X is given by:

$$p(k) = P(X = k) = \frac{\lambda^k}{k!}e^{-\lambda}, \quad k = 0, 1, 2, \ldots$$

where $\lambda > 0$.

- **Poisson Process**. Let $\lambda > 0$ be fixed. Let $N(t)$ be a random variable denoting the number of events that occur from time $t = 0$ up to time t. $N(t)$ is a Poisson process with rate λ if the following conditions hold:
  - $N(0) = 0$.
  - $N(t)$ has independent increments.
  - The number of "arrivals" in any interval of length $\tau > 0$ is Poisson($\lambda\tau$) distributed.

- **Monotonic Transformations**: Let X be a random variable with continuous pdf $f_X(x)$ on $\mathcal{X}$, and let $Y = g(X)$. If g is a strictly monotonic transformation such that g^{-1} has a continuous derivative on $\mathcal{Y}$, then the pdf of Y is given by

$$f_Y(y) = \begin{cases} f_X\left(g^{-1}(y)\right)\left|\frac{d}{dy}g^{-1}(y)\right| & y \in \mathcal{Y} \\ 0 & \text{otherwise.} \end{cases}$$

  - Note that if g is *not* strictly monotonic, this formula cannot be directly applied.
  - When this formula does not work, consider using the "CDF-method".
  - You can also extend this formula by breaking the function g into parts that are strictly monotonically increasing or decreasing, and then summing the functions. When doing so, be careful with the domain of the functions.