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Homework 2

1. (from last HW, 3 points) Suppose that X is a discrete random variable with:

$$P(X = x) = \begin{cases} \frac{2}{3}\theta & x = 0 \\ \frac{1}{3}\theta & x = 1 \\ \frac{2}{3}(1-\theta) & x = 2 \\ \frac{1}{3}(1-\theta) & x = 3 \end{cases}$$

   where $0 \leq \theta \leq 1$. Suppose we observe 10 independent observations from such a distribution: $(3, 0, 2, 1, 3, 2, 1, 0, 2, 1)$. (We'll return to this same distribution later)

   - (2 points) Find the MLE of θ.
   - (1 point) Find an approximate standard error for your estimate.

2. Often, real data is messy, and may have missing information. In the following examples, we consider finding the MLE in these types of scenarios.

   (a) (2 points) We believe a particular measurement is approximately normally distributed. However, we are only able to measure accuracy up to integer values. Thus, we observe 3 partial values: $1 \leq x_1^* < 2$, $3 \leq x_2^* < 4$, and $6 \leq x_3^* < 7$, only knowing the interval into which the observations fall rather than the specific value. Find an expression of the likelihood function under this model (Hint: You can use the $\Phi(x)$ notation for the CDF of a standard normal.)

   (b) (3 points) Suppose 100 seeds are planted, and it is known only that $x^* \leq 10$ seeds germinated—the exact number of germinating seeds is unknown. Let θ be the probability that seed i germinates (the ith seed will either germinate, or it will not), and we assume that seeds are independent. What is the likelihood function of this model? Find a maximum likelihood estimate. Plot the likelihood function, and comment about the point estimate (the MLE) and the likelihood function.

3. (3 points) **Ecology: capture / recapture**. A common approach to estimate the number of animals N in a given population is the following: Capture a subset of the population, mark them with tags and release them into the wild. After some time, we recapture a sample from the population of size N, and calculate the proportion of re-captured animals that are marked (previously captured), and use this to estimate the total population size.

   Suppose we want to estimate the number of badgers N in a region. We capture and tag $N_1 = 25$ animals, and release them. Later, we capture $n = 60$ badgers, and we find $n_1 = 5$ tagged animals, and $n_2 = 55$ non-tagged animals. Assuming the badgers were caught randomly, use a hypergeometric distribution to get find the likelihood, as a function of $\theta = N$. Find an MLE, and plot plot the likelihood function.