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Homework 3

Note: The first two problems are distinct from what we have seen so far. Here, the support of X_i depends on θ. Think carefully about the MLE in these situations.

1. (3 points) Suppose $X_1, X_2, \ldots, X_n$ are iid from a Uniform$(0, \theta)$ distribution. What is the MLE of θ?

2. (4 points) Suppose that $X_1, \ldots X_n$ are iid from a distribution with the density function:

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta \\ 0, & \text{otherwise} \end{cases}$$

   (a) (2 points) Find the method of moments estimate of θ.

   (b) (2 points) Find the MLE of θ (*HINT:* be careful about this one. What values of θ gives a positive likelihood?)

3. (3 points) Suppose we observe a collection of observations, (X_1, Y_1), $(X_2, Y_2), \ldots (X_n, Y_n)$. We're particularly interested in a model for $Y|X$, conditioning on the observed X. The model we would like to fit is

$$Y_i = f_\theta(X_i) + \epsilon_i,$$

   where ϵ_i is an error term, and f_θ is some function of input data X_i and parameters θ. In practice, this function is fixed to be within a particular family (i.e., a neural network, linear model, etc.)

   One way to estimate θ with this model is minimizing the mean-square-error:

$$\hat{\theta} = \text{argmin}_\theta \frac{1}{n} \sum_{i=1}^{n} \big(Y_i - f_\theta(X_i)\big)^2.$$

   Show that this is the same answer that you get by fitting the MLE to a conditional normal distribution:

$$Y_i | X_i \overset{iid}{\sim} N\big(f_\theta(X_i), \sigma^2\big).$$

   For convenience, you can assume that σ^2 is fixed, and we want to estimate θ. (*Note:* This is also the same as assuming that the ϵ_i are iid normal)

4. (4 points) For this problem, you will implement logistic regression from scratch. The math was worked out in class in the Maximum Likelihood slides and notes; the intent is to build this from scratch rather than using pre-built logistic regression functions, though they will be useful if you want to check your solution.

   The data you should estimate is called `mtcars`. It is pre-built in R and can be access simply by typing `mtcars`. A `.csv` will also be uploaded to Canvas. Your task is to predict $Y_i =$ `am`, a binary variable in the data indicating whether or not the car is an automatic (0) or manual (1) based on other car features, X_i: `mpg, wt, disp, cyl`.

   More information about the data can be found in R by running the command `?mtcars`.

   To show your work, upload your code and provide some details such as parameter estimates or predictive accuracy.