Stat 5200: Analysis of Designed Experiments

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Course Introduction

Course Overview

- This class is all about developing techniques for answering scientific questions.
  - Is a newly proposed drug a safe and effective cure for a disease?
  - Which combination of protein and carbohydrate sources provide the best nutrition for growing lambs?
  - How will long-distance telephone usage change if our company offers a different rate structure to our customers?
- Our goal is to help decision makers and researchers design good experiments, analyze them properly, and answer their research questions.
- The course is based on the textbook "A First Course in Design and Analysis of Experiments" (Oehlert2000).

Course Notation

Useful course notation

Basic notations we will use throughout the class:

- y_{ij}
- $j : k$, $y_{j:k}$, etc.
- $y_{\cdot j}$ and $y_{i\cdot}$
- $\bar{y}$, $\bar{y}_{i\cdot}$, etc.
- y_{ij}^*
- $\mathbb{R}$

Experiments

What is an "Experiment" and why it matters

- Experiments are the gold-standard for scientific inference.
- Origins of the design a study of experiments starting with R. A. Fisher in the 1920s–1940s at UK agricultural center.
- His work developed the field of experimental design, which is used now in all forms of scientific investigations.
- Large advances in medicine, science, and product quality improvements can be attributed to proper experimental design and analysis.

What is an "Experiment" and why it matters II

Fisher's early examples

Fisher found decades worth of fertilizer treatment data:

- Fertilizer had been applied to a field one year, and not the next, in order to compare yield of grain produced in the two different years.
- Fertilizer was applied to one field in one year, and not to a nearby field in the same year, again to compare the yield of grain produced in the same year.

What is an "Experiment" and why it matters III

From a statistical perspective, there are three components that define an experiment: Treatments, Experimental Units, and Assignment Method.

Experiment

An experiment is specified by the combination of treatments, experimental units, and assignment methods.

- Note: There is no mention of a *method* for analyzing the results of an experiment. Strictly speaking, the analysis is not part of the design, though a wise experimenter will consider the analysis when planning an experiment.

Treatments

Experimental Units

Assignment Methods

Measurement Units vs Experimental Units

- A common source of difficulty and confusion is the distinction between *experimental units* and *measurement units*.

Definition: Measurement unit

A measurement unit is the individual or piece of experiment material you actually measure.

- Consider an educational study, where 6 classrooms of first graders are assigned at random to two different reading programs, and all first graders are later evaluated via a common reading exam at the end of the year.

Measurement Units vs Experimental Units II

- In an animal nutrition experiment, different types of diets are given to pens that each contain many individual animals.

High-level and Low-level EU's

- The distinction between experimental units and measurement units can become complicated.
- For example, in split-plot experiments (discussed later), there can be multiple layers of comparisons, with two or more types of experimental units with low-level EUs, serving as measurement units for the high-level EUs.

(Example on next slide)

High-level and Low-level EU's II

Baking experiment

A baking experiment is used to study the effect of high and low levels of three ingredients in a recipe ($2^3 = 8$ possible combinations), and the effect of baking temperature (6 different levels). You have an oven that can bake 8 batches at one time, at one temperature.

In the experiment, you run the oven 6 times, each with 8 batches made up of the 8 combinations of ingredients randomly placed in the oven.

What are the treatments, experimental units, and measurement units?

Observational studies vs experiments

Observational studies vs experiments

- Experiments help answer questions, but there are also non-experimental techniques. What makes experiments special?
- The most important feature is that an experiment gives researchers control: we get to pick the treatment and assignments.
- This is different from *observational studies*, where there are also treatments, units, and responses, but the researcher does not get to control treatment assignment.

Observational studies vs experiments II

- If an experiment is designed well, we can
  - Directly compare impact of various treatments, minimizing *confounding*.
  - Design experiments to minimize any form of *bias*.
  - Design experiments so that the *error* in comparison is small.

Design considerations

Design issues and considerations

- An important feature of an experiment is that the units are independent, meaning that the response of one unit is *not* influenced by treatments or responses of other units.
- We should also consider edge effects, which can occur in both space and time:
  - **EX (Space):** Plants at the edge of a plot or a greenhouse may respond differently to treatments.
  - **EX (Time):** Flushing out old treatments, or applying treatments at certain time of day.
  - We should account for these possibilities by ensuring sufficient separation of units, and provisioning a "buffer" in both space and time.

Fixed resource problem

- An often-encountered design problem is when there is a fixed amount of resource available for experimenting.
- Examples:
  - Fixed sized field
  - Fixed amount of time
  - Fixed amount of money to spend on experiment.
- In these situations, a key part of the problem include: Size / shape of plots, number of samples per session, etc.
- Useful rule of thumb: Do *more*, less well. That is, have more experimental units instead of more measurement units.

Other design considerations

Open to the class: **What issues may arise that you should consider in an experimental design?**

References and Acknowledgements

Oehlert GW (2000). *A First Course in Design and Analysis of Experiments*. W. H. Freeman. ISBN 0-7167-3510-5.

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